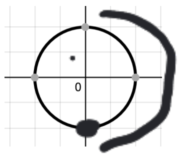
MATH 8 – Sample Final

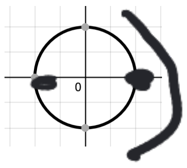
This test is in two parts. On part one, you may not use a calculator; on part two, a calculator is necessary. When you complete part one, you turn it in and get part two. Once you have turned in part one, you may not go back to it.

PART ONE - NO CALCULATORS ALLOWED

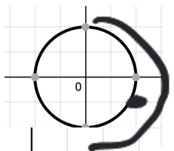
(1) Find each of the following:
(Note: answers to inverse trig. problems should be in radians, not degrees)



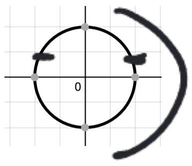
(a) $\sin^{-1}(-1) = \frac{-\pi}{2}$



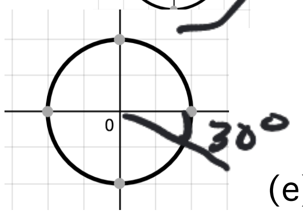
(b) $\tan^{-1}(0) = 0$



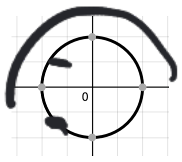
(c) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \frac{-\pi}{6}$



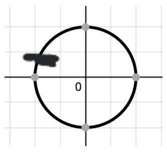
(d) $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$



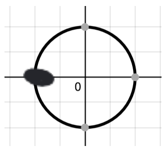
(e) $\tan 330° = -\frac{1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$



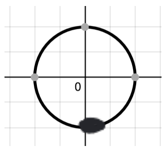
(f) $\cos^{-1}\left(\frac{-\sqrt{2}}{2}\right) = \frac{3\pi}{4}$



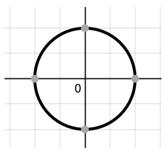
(g) $\sec\left(\frac{5\pi}{6}\right) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$



(h) $\csc(\pi) =$ undefined



i) $\cos^{-1}\left(\underbrace{\cos\left(\frac{3\pi}{2}\right)}_{0}\right) = \frac{\pi}{2}$



(j) $\tan\left(\tan^{-1}(1/3)\right) = 1/3$

(2) Fill in the blank to complete the identity.

(a) $\sin 2\theta = 2\sin\theta\cos\theta$

(b) $\cos^2 x = 1-\sin^2\theta$ or $\frac{1+\cos 2\theta}{2}$

(c) $\sin(\theta/2) = \pm\sqrt{\frac{1-\cos 2\theta}{2}}$

(d) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

MATH 8 – Sample Final Exam – Part Two

Fill in the blanks. In problems 1 - 7 fill in the blank with the most appropriate answer

(1) True or false $\sin(-\theta) = -\sin(\theta)$ true

sine and tangent odd functions, cosine is even

(2) In what quadrant(s) is $f(x) = \cot(x) > 0$ 1, 3

(3) $\begin{vmatrix} -5 & 3 \\ 2 & -7 \end{vmatrix} =$ 29

(4) The period of $f(x) = \tan(3\pi x)$ is $1/3$ $\quad \frac{\pi}{3\pi}$

(5) The range of $f(x) = \sin^{-1}(x)$ is $[-\pi/2, \pi/2]$

(6) To graph $f(x) = 2\sin(4x - \pi)$ we would shift the graph of $g(x) = 2\sin(4x)$ how far to the right $\pi/4$

$\sin 4(x - \frac{\pi}{4})$

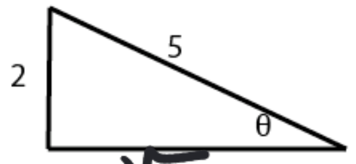
(have to factor out the 4.

(7) The range of f $f(x) = \tan(x)$ is $(-\infty, \infty)$

(8) How many solutions does the equation $\sin(\theta) = -0.2$ have for $0 \le \theta \le \pi$ none

3 ways of defining trig function

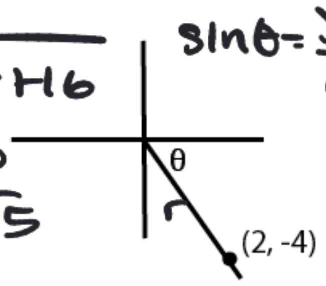
(9) Given the following figures, find:



$\sqrt{21}$

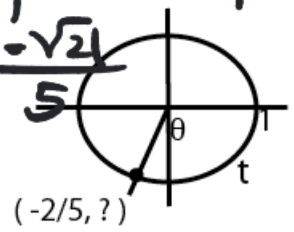
$r = \sqrt{4+16}$
$= \sqrt{20}$
$= 2\sqrt{5}$

$\sin\theta = \frac{y}{r}$



$(-\frac{2}{5})^2 + y^2 = 1$

$y = \frac{-\sqrt{21}}{5}$



(a) cos θ = $\frac{\sqrt{21}}{5}$ (c) sin θ = $\frac{-4}{2\sqrt{5}} = \frac{-2}{\sqrt{5}}$ (e) sin t = $-\frac{\sqrt{21}}{5}$

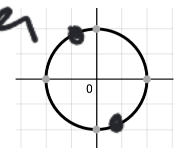
(b) $\theta \approx$ 23.6 degrees (d) $\theta \approx$ -63.4 degrees (f) $\theta \approx$ -113.6 degrees

Right △ point on term. side point on unit circle

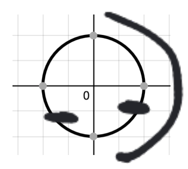
(10) Solve the following equations for the given restriction on t. (If no restriction is given, find all solutions)

(a) Solve: $\csc(t) = -2$ for $0 \le t < 2\pi$ $t = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\quad \sin t = -\frac{1}{2}$

(b) Solve: $\tan(t) = -\sqrt{3}$ $t = \frac{2\pi}{3} + \pi k$, k integer



(c) Solve: $\sin(t) = -\frac{\sqrt{2}}{2}$ for $-\frac{\pi}{2} \le t < \frac{\pi}{2}$ $t = \frac{-\pi}{4}$



(11) Given the following matrices:

$$A=\begin{bmatrix} 2 & -1 \\ 3 & -5 \end{bmatrix} \quad B=\begin{bmatrix} 3 & 1 & 5 \\ 0 & 4 & 3 \\ 1 & -2 & 3 \end{bmatrix} \quad C=\begin{bmatrix} 1 & -3 \\ 3 & 7 \end{bmatrix}$$

Find the following, if possible. (If not possible, say so.)

(a) AB

$\boxed{\text{not possible}}$

(b) AC

$\boxed{\begin{bmatrix} -1 & -13 \\ -12 & -44 \end{bmatrix}}$

(e) det(B)

$3\begin{vmatrix} 4 & 3 \\ -2 & 3 \end{vmatrix} + 1\begin{vmatrix} 1 & 5 \\ 4 & 3 \end{vmatrix}$

$= 3(18) + -17 = \boxed{37}$

(12) SOLVE the following equations: $0 \le x < 2\pi$

(a) sin2x = 3sinx

$2\sin x\cos x = 3\sin x$ *

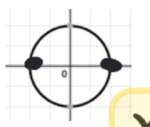
$2\sin x\cos x - 3\sin x = 0$

$\sin x(2\cos x - 3) = 0$

$\sin x = 0 \qquad 2\cos x - 3 = 0$

$\cos x = 3/2$

no soln since $-1 \le \cos x \le 1$



$\boxed{x = 0, \pi}$

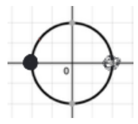
(*) why can't we divide out or "cancel" sinx here?

(b) $\cos^2(3x) - 1 = 0$

$\cos^2(3x) = 1$ remember +/-

$\cos(3x) = \pm 1$

special situation, multiple angle argument in $[0, 2\pi]$



$3x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$

go around 3 times

$\boxed{x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}}$

(OR All solns $3x = \pi k$

$x = \frac{\pi}{3}k$ and generate)

(13) Given $\csc\alpha = -5/4$, $\pi < \alpha < \frac{3\pi}{2}$, and $\beta = \sin^{-1}(2/3)$,

Find:

$\pi < \alpha < \frac{3\pi}{2}$

$\frac{\pi}{2} < \frac{\alpha}{2} < \frac{3\pi}{4} \Rightarrow$ Q2

$\sin(\alpha/2) > 0$

a) $\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 - -3/5}{2}} = \sqrt{\frac{5+3}{10}} = \boxed{\sqrt{\frac{4}{5}}}$

b) $\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2\left(\frac{2}{\sqrt5}\right)}{1-\frac{4}{5}} = \frac{4/\sqrt5}{1/5}$

$= \frac{4}{\sqrt5}\cdot 5 = \frac{20}{\sqrt5}$

$= \boxed{4\sqrt5}$

c) $\cos(\alpha + \beta)$

$\cos\alpha\cos\beta - \sin\alpha\sin\beta$

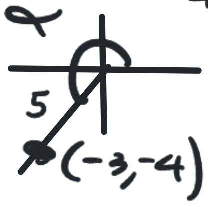
$= -\frac{3}{5}\cdot\frac{\sqrt5}{3} - -\frac{4}{5}\cdot\frac{2}{3} = \boxed{\frac{8-3\sqrt5}{15}}$

$\csc\alpha = -\frac{5}{4} = \frac{r}{y}$

$r > 0$

$r = 5$

$y = -4$



$x^2 + y^2 = r^2$

$x^2 + 16 = 25$

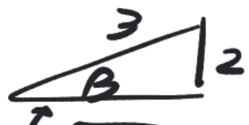
$x = \pm 3$

Q3 $\Rightarrow x = -3$

$\beta = \sin^{-1} 2/3$

$\sin\beta = 2/3$

Q1



$\sqrt{9-4}$

$\sqrt5$

(14) Verify the identity : $\dfrac{1-\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1-\sin\theta}=2\sec\theta$

Rewrite original LHS

: $\dfrac{1-\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1-\sin\theta} = \dfrac{1-\sin\theta}{\cos\theta}\cdot\dfrac{1-\sin\theta}{1-\sin\theta} + \dfrac{\cos\theta}{1-\sin\theta}\cdot\dfrac{\cos\theta}{\cos\theta} = \dfrac{(1-\sin\theta)^2+\cos^2\theta}{(1-\sin\theta)\cos\theta}$

LCD: $\cos\theta(1-\sin\theta)$

Build fractions to have LCD

$= \dfrac{1-2\sin\theta+\sin^2\theta+\cos^2\theta}{(1-\sin\theta)(\cos\theta)}$

$= \dfrac{1-2\sin\theta+1}{(1-\sin\theta)\cos\theta}$

$= \dfrac{2-2\sin\theta}{1-\sin\theta\ \cos\theta}$

$= \dfrac{2\cancel{(1-\sin\theta)}}{\cancel{(1-\sin\theta)}\cos\theta}$

$= \dfrac{2}{\cos\theta} = 2\sec\theta$

so $\dfrac{1-\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1-\sin\theta}=2\sec\theta$

(15) Sketch the following graph. (clearly show scale, graph at least one p... coordinates of highs and lows)

$f(x)=2\sin\left(3x-\dfrac{\pi}{4}\right) = 2\sin\left(3\left(x-\dfrac{\pi}{12}\right)\right)$

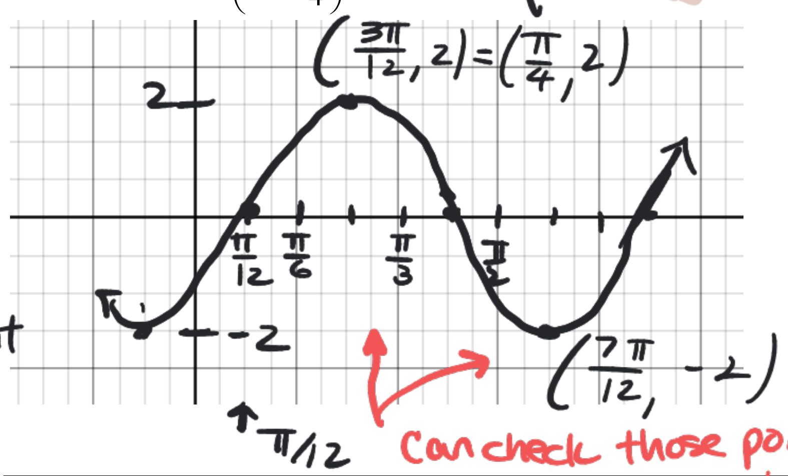
Amp = 2

period $\dfrac{2\pi}{3}$

$\frac{1}{4}$ period

$\frac{1}{4}\cdot\frac{2\pi}{3}=\frac{\pi}{6}$

shift Right $\dfrac{\pi}{12}$



Can check those points in equation

(16) Use Gaussian Elimination to solve:
(no credit if requested method is not used)

$$\begin{cases}3x-y-z=8\\x+y-2z=5\\2x-y+z=1\end{cases}$$

get 1 $\quad R_1 \leftrightarrow R_2 \quad$ use 1 to get zeros below $\quad -3R_1+R_2\to R_2,\ -2R_1+R_3\to R_3 \quad$ get 1

$$\left[\begin{array}{ccc|c}3&-1&-1&8\\1&1&-2&5\\2&-1&1&1\end{array}\right]\Rightarrow\left[\begin{array}{ccc|c}1&1&-2&5\\3&-1&-1&8\\2&-1&1&1\end{array}\right]\Rightarrow\Rightarrow\Rightarrow\left[\begin{array}{ccc|c}1&1&-2&5\\0&-4&5&-7\\0&-3&5&-9\end{array}\right]$$

$-\frac{1}{4}R_2\to R_2 \quad$ use 1 to get zeros below $\quad 3R_2+R_3\to R_3 \quad \frac{4}{5}R_3\to R_3$

$$\left[\begin{array}{ccc|c}1&1&-2&5\\0&1&-5/4&7/4\\0&-3&5&-9\end{array}\right]\Rightarrow\Rightarrow\Rightarrow\left[\begin{array}{ccc|c}1&1&-2&5\\0&1&-5/4&7/4\\0&0&5/4&-15/4\end{array}\right]$$

$2R_3+R_1\to R_1$, $\frac{5}{4}R_3+R_2$ $\quad -R_2+R_1\to R_1$

$$\left[\begin{array}{ccc|c}1&1&-2&5\\0&1&-5/4&7/4\\0&0&1&-3\end{array}\right]\Rightarrow\left[\begin{array}{ccc|c}1&1&0&-1\\0&1&0&-2\\0&0&1&-3\end{array}\right]\to\left[\begin{array}{ccc|c}1&0&0&1\\0&1&0&-2\\0&0&1&-3\end{array}\right]$$

use 1 to get zeros above

use 1 to get zero above

$\boxed{(1,-2,-3)}$

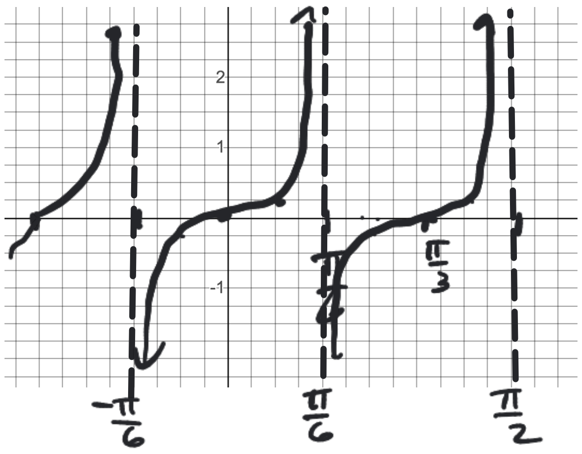
(17)
Sketch the following graph. (clearly show scale, graph at least TWO periods, show location of any asymptotes, label 2 points on graph) (5 points)

$f(x)=\frac{1}{4}\tan(3x)$

period $= \frac{\pi}{3}$

Asymptote when $3x=\frac{\pi}{2}$

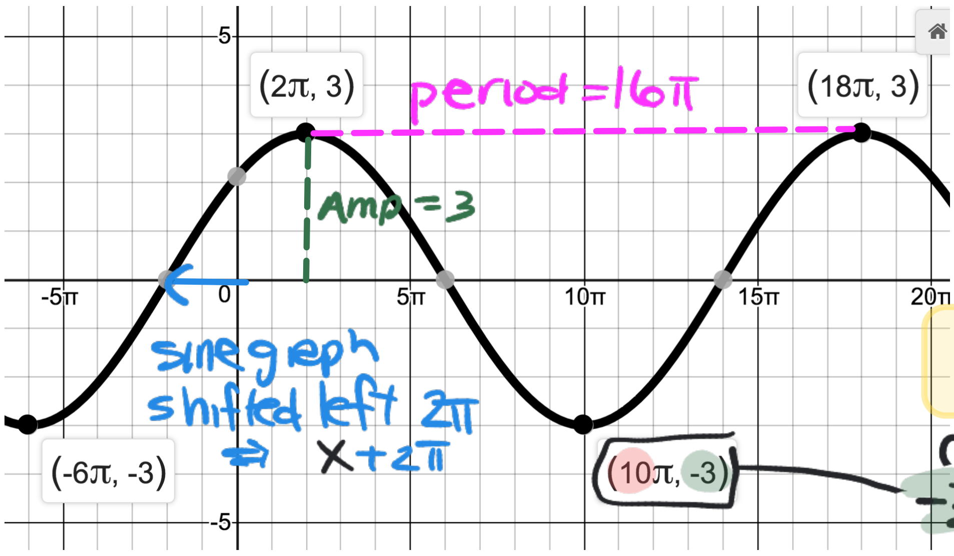
$x=\frac{\pi}{6}$



(18) Find an equation corresponding the graph below. Check a point.

period: $\frac{2\pi}{K}=16\pi$

$\Rightarrow K=\frac{1}{8}$



$y=3\sin\left(\frac{1}{8}(x+2\pi)\right)$

$\boxed{y=3\sin\left(\frac{1}{8}x+\frac{\pi}{4}\right)}$

check point

$-3 \overset{?}{=} 3\sin\left(\frac{1}{8}(10\pi)+\frac{\pi}{4}\right)$

$3\sin\left(\frac{5\pi}{4}+\frac{\pi}{4}\right)$

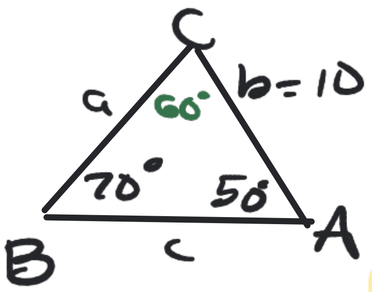
$3\sin\left(\frac{6\pi}{4}\right)$

$=3\sin\left(\frac{3\pi}{2}\right)$

$=3(-1)$

$=-3$

(19) Given triangle ABC with A=50°, B=70° and b=10 inches, find the remaining parts.



Find C $70°+50°+C=180°$

$C=60°$ ✓

$\frac{c}{\sin 60°}=\frac{10}{\sin 70°}=\frac{a}{\sin 50°}$

$\boxed{c=\frac{10\sin 60°}{\sin 70°}}$ $\boxed{a=\frac{10\sin 50°}{\sin 70°}}$

Find all solutions to the following equations.

(21) $3\tan^2 x - \sec^2 x - 5 = 0$

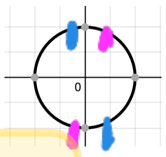
identity $\sec^2 x = \tan^2 x + 1$

$3\tan^2 x - (\tan^2 x + 1) - 5 = 0$

$2\tan^2 x - 6 = 0$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$



$\boxed{x = \frac{2\pi}{3} + \pi k,\ \frac{\pi}{3} + \pi k}$

(22) $\cos(2x) = 2 + 5\cos x$

identity $\cos 2x = 2\cos^2 x - 1$

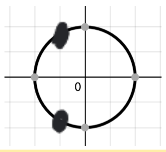
$2\cos^2 x - 1 = 2 + 5\cos x$

$2\cos^2 x - 5\cos x - 3 = 0$

$(2\cos x + 1)(\cos x - 3) = 0$

$2\cos x + 1 = 0$

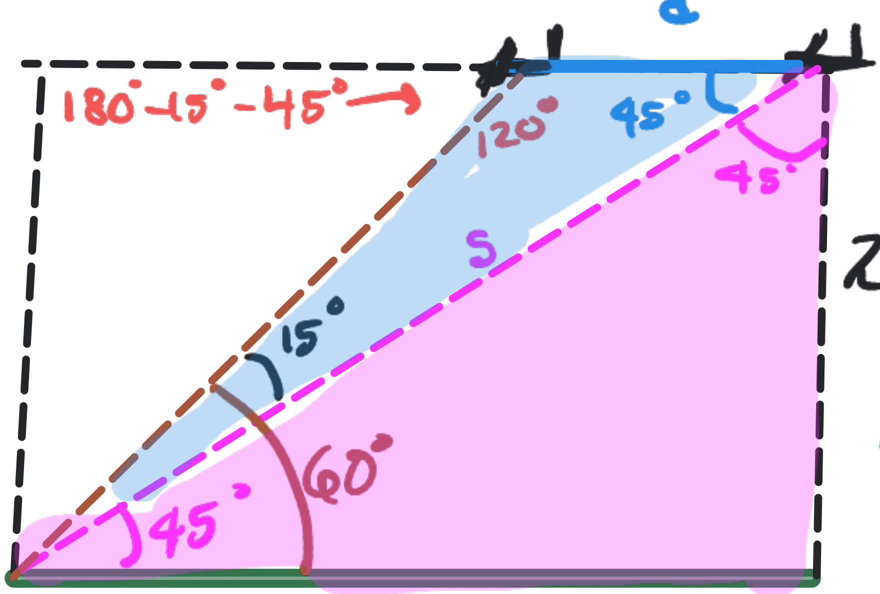
$\cos x = -1/2$



$\cos x = 3$ no soln since $-1 \le \cos x \le 1$

$\boxed{x = \frac{2\pi}{3} + 2\pi k,\ \frac{4\pi}{3} + 2\pi k}$

(23) A man looks up and sees an airplane flying in his direction at a level altitude of 2 miles. He watches the airplane for a few minutes. During that period of time he notices that the angle of elevation to the airplane changes from 45° to 60°. How far has the plane traveled in that time?



2 miles

Many approaches. Start by putting in angles which can be found using geometry.

From right Δ

$\frac{opp}{hyp}$ $\frac{2}{s} = \sin 45°$

$s = \frac{2}{\sin 45°} = \frac{2}{\sqrt{2}/2} = \frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

Now using Law of Sines

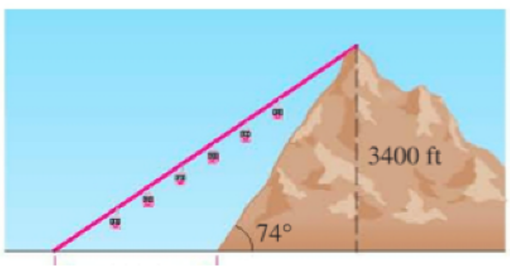
$\frac{d}{\sin 15°} = \frac{s}{\sin 120°} \Rightarrow \frac{d}{\sin 15°} = \frac{2\sqrt{2}}{\sqrt{3}/2}$

$d = \frac{2\sqrt{2}\sin 15°}{\sqrt{3}/2}$

$= \boxed{\frac{4\sqrt{2}\sin 15°}{\sqrt{3}} \approx .845 \text{ mi}}$

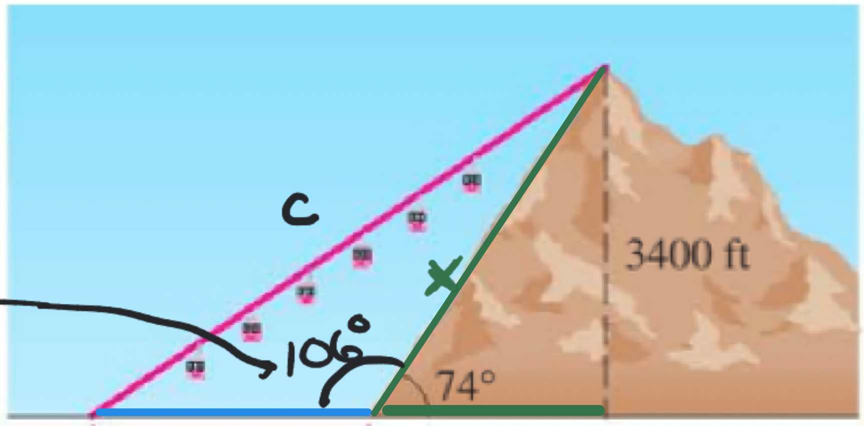
(24) You do not need the exact answer, but your answer must match the exact answer to 3 decimal places.

Cable Car A steep mountain is inclined 74° to the horizontal and rises 3400 ft above the surrounding plain. A cable car is to be installed from a point 800 ft from the base to the top of the mountain, as shown. Find the shortest length of cable needed.



Cable Car A steep mountain is inclined 74° to the horizontal and rises 3400 ft above the surrounding plain. A cable car is to be installed from a point 800 ft from the base to the top of the mountain, as shown. Find the shortest length of cable needed.

$180° - 74°$



800ft

Can use Law of Cosines on blue △

$$c^2 = 800^2 + x^2 - 2\cdot 800\cdot x\cos 106°$$

But need x.

Find x using right △

$$\sin 74° = \frac{3400}{x}$$

$$x = \frac{3400}{\sin 74°}$$ store in calculator memory

$$c^2 = 800^2 + x^2 - 2\cdot 800\cdot x\cos 106°$$

$$c = \sqrt{800^2 + x^2 - 2\cdot 800\cdot x\cos 106°}$$

$$c \approx 3835.41286049 \text{ ft}$$

Just so you can check your calculator accuracy.

Notice - I didn't need to write down any intermediate values.